Hand in at beginning of next lecture

For these problems you should use the following facts

\[ \int_0^\infty dx e^{-ax} = \frac{1}{\alpha} \]  
\[ \int_0^\infty dx xe^{-ax} = \frac{1}{\alpha^2} \]  
\[ \int_0^\infty dx x^2 e^{-ax} = \frac{2}{\alpha^3} \].

We are going to use the variational method to estimate the zero point energy of the harmonic oscillator. We will minimize

\[ E = \int dx \left[ -\frac{\hbar^2}{2m} \psi'(x)\psi''(x) + \frac{1}{2} m\omega^2 x^2 \psi^*(x)\psi(x) \right]. \]  

Life is a bit easier if we integrate by parts and write

\[ E = \int dx \left[ \frac{\hbar^2}{2m} \psi^*(x)\psi'(x) + \frac{1}{2} m\omega^2 x^2 \psi^*(x)\psi(x) \right]. \]

We will use the ansatz

\[ \psi(x) = \frac{e^{-|x|/(2d)}}{\sqrt{2d}}. \]

**Problem 1.** Calculate \( \int |\psi(x)|^2 dx \).

**Solution 1.1.**
Problem 2. Calculate $\int \psi'^*(x)\psi'(x) \, dx$.

Solution 2.1.

Problem 3. Calculate $\int x^2|\psi(x)|^2 \, dx$.

Solution 3.1.
Problem 4. Put these results together to calculate $E$.

Solution 4.1.

Problem 5. Minimize $E$. (It might help to first plot it)

Solution 5.1.