Hand in at beginning of next lecture

Problem 1. Constituent Equation
In 2+1D electromagnetism there are three physical fields

\[
S_t = F_{xy} = \partial_x A_y - \partial_y A_x
\]

\[
S_y = F_{tx} = \partial_t A_x - \partial_x A_t
\]

\[
S_x = F_{yt} = \partial_y A_t - \partial_t A_y.
\]

Show that

\[
\partial_t S_t + \partial_x S_x + \partial_y S_y = 0,
\]

Solution 1.1.

Problem 2. Light in 2D

2.1. In the absence of matter, the 2D Maxwell equations read

\[
\partial_x S_y - \partial_y S_x = 0
\]

\[
-\partial_y S_t - \partial_t S_y = 0
\]

\[
\partial_t S_x + \partial_x S_t = 0
\]

\[
\partial_t S_t + \partial_x S_x + \partial_y S_y = 0,
\]
Make the ansatz

\[ S_\nu(x, y, t) = S_\nu e^{-i\omega t + ik_x x + ik_y y} \]  

to find the dispersion relationship.

Solution 2.1.

2.2. For a given \( \vec{k} \), how many modes are there? What is the physical interpretation?

Solution 2.2.
2.3. A somewhat more complicated way to do the same calculation is to work with the potentials $A_x, A_y, A_t$. This more complicated approach will come in handy on Thursday. Write Eq. (5) in terms of $A$.

Solution 2.3.

2.4. Make the ansatz

$$A_\nu(x, y, t) = A_\nu e^{-i\omega t + ik_x x + ik_y y},$$

and fill in the boxes below.

Solution 2.4. .

$$\begin{pmatrix}
\omega^2 - k_y^2 & \square & \square \\
\square & \omega^2 - k_x^2 & \square \\
\square & \square & k_x^2 + k_y^2
\end{pmatrix}
\begin{pmatrix}
A_x \\
A_y \\
A_t
\end{pmatrix} = 0$$
2.5. Show that
\[
\begin{pmatrix}
-k_y \\
-k_x \\
0
\end{pmatrix}
\] (11)
is an eigenvector of this matrix. What is its eigenvalue?

Solution 2.5.


Solution 2.6.

2.7. Show that
\[
\begin{pmatrix}
k_x \\
0 \\
k_y \\
-\omega
\end{pmatrix}
\] (12)
is an eigenvector of this matrix. What is its eigenvalue?

Solution 2.7.


Solution 2.8.
2.9. Show that
\[
\begin{pmatrix}
k_x \omega \\
k_y \omega \\
k^2
\end{pmatrix}
\] (13)
is an eigenvector of this matrix. What is its eigenvalue?

Solution 2.9.

2.10. Can this eigenvalue ever be zero (for real \( \omega \))?

Solution 2.10.