Scalar QED3 Reference Sheet

$$S_t = F_{xy} = \partial_x A_y - \partial_y A_x$$

Magnetic field

$$S_y = F_{tx} = \partial_t A_x - \partial_x A_t$$
$$S_x = F_{yt} = \partial_y A_t - \partial_t A_y.$$

Electric field

$$(\partial_t - iA_t)^2\phi - (\partial_x - iA_x)^2\phi - (\partial_y - iA_y)^2\phi + m^2\phi = 0.$$

Klein-Gordon field

Maxwell's Equations

$$\partial_x S_y - \partial_y S_x = \frac{\phi^*(\partial_t - iA_t)\phi - \phi(\partial_t + iA_t)\phi^*}{2i} \quad \longleftarrow \quad \nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$-\partial_y S_t - \partial_t S_y = -\frac{\phi^*(\partial_x - iA_x)\phi - \phi(\partial_x + iA_x)\phi^*}{2i}$$
$$\partial_t S_x + \partial_x S_t = -\frac{\phi^*(\partial_y - iA_y)\phi - \phi(\partial_y + iA_y)\phi^*}{2i}$$

$$\longleftarrow \quad \nabla \times B - \epsilon_0 \partial_t E = \mu_0 J$$

$$\partial_t S_t + \partial_x S_x + \partial_y S_y = 0, \quad \longleftarrow \quad \nabla \times E = -\partial_t B.$$

Maxwell's Equations in terms of potentials

$$\partial_t[\partial_x A_x + \partial_y A_y] - (\partial_x^2 + \partial_y^2)A_t = \frac{\phi^*\partial_t\phi - \phi\partial_t\phi^*}{2i} - |\phi|^2 A_t$$
$$\partial_x[-\partial_y A_y + \partial_t A_t] - (-\partial_y^2 + \partial_t^2)A_x = -\frac{\phi^*\partial_x\phi - \phi\partial_x\phi^*}{2i} + |\phi|^2 A_x$$
$$\partial_y[\partial_t A_t - \partial_x A_x] - (\partial_t^2 - \partial_x^2)A_y = -\frac{\phi^*\partial_y\phi - \phi\partial_y\phi^*}{2i} + |\phi|^2 A_y$$

Linearized equations for Gauge Fields

$$\begin{pmatrix} \omega^2 - k_y^2 & k_x k_y & \omega k_x \\ k_x k_y & \omega^2 - k_x^2 & \omega k_y \\ \omega k_x & \omega k_y & k_x^2 + k_y^2 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_t \end{pmatrix} = 0$$

Linearized equations for Gauge Fields in Symmetry broken phase

$$\begin{pmatrix} \omega^2 - k_y^2 - \phi_0^2 & k_x k_y & \omega k_x & i k_x \phi_0 \\ k_x k_y & \omega^2 - k_x^2 - \phi_0^2 & \omega k_y & i k_y \phi_0 \\ \omega k_x & \omega k_y & k_x^2 + k_y^2 + \phi_0^2 & i\omega\phi_0 \\ -i k_x \phi_0 & -i k_y \phi_0 & -i\omega\phi_0 & \omega^2 - k^2 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_t \\ b \end{pmatrix} = 0$$