PHYS 3317: Classical Expansion Dynamics
Lecture 2

Name/NetID: ____________________

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Hand in at beginning of next lecture

1. **Particle trajectories**

   (a) Imagine I have a collection of classical particles moving in one dimension, labelled by $j = 1, 2, \cdots N$. At time $t = 0$ they are all at the origin, $x_j(t = 0) = 0$, and you know their momenta $p_j(t = 0) = p_j^{(0)}$. What is $p_j(t)$?

   (b) What is $x_j(t)$?

   (c) In thermodynamics and fluid mechanics, it is traditional to not specify the position and momentum of each particle, but instead to specify a distribution $f(x, p, t)$: $f(x, p, t)dx\,dp$ is the number of particles within dx of position x and within dp of momentum p. Formally one can write

   $$f(x, p, t) = \sum_j \delta\left(x - x_j(t)\right)\, \delta\left(p - p_j(t)\right), \tag{1}$$

   though in practice we smear out the delta-functions to make a continuous distribution. Using your previous results, what is $f(x, p, t)$ in terms of a sum over delta functions?

(d) If we throw away momentum information, we get the density

$$n(x,t) = \sum_j \delta\left(x - x_j(t)\right). \tag{2}$$

Putting in our explicit formulae for $x_j(t)$, what is $n(x,t)$ in terms of a sum over delta functions involving $p_j(0)$?

(e) If we throw away position information, we get the momentum-space density

$$n_k(p,t) = \sum_j \delta\left(p - p_j(t)\right). \tag{3}$$

What is $n_k(p,t)$ in terms of a sum over delta functions involving $p_j(0)$?

(f) Using that $\delta(ax) = \delta(x)/a$, relate $n(x,t)$ to $n_k(p,0)$.