PHYS 3317: Josephson Oscillations

Tuesday, Nov. 25, 2014

Hand in at beginning of next lecture

In this problem you will model two superconductors connected by a tunnel barrier. We are modelling the "Cooper pair wavefunction" by an equation

$$i\partial_t\psi_L = \gamma(|\psi_L|^2 - \rho)\psi_L - J\psi_R \quad (1)$$

$$i\partial_t\psi_R = \gamma(|\psi_R|^2 - \rho)\psi_R - J\psi_L. \quad (2)$$

Problem 1. Josephson Oscillations

1.1. These equations are simpler if one transforms into "polar" coordinates. Make the substitution

$$\psi_L = f_L e^{i\theta_L} \quad (3)$$

$$\psi_R = f_R e^{i\theta_R}. \quad (4)$$

Substitute these into Eq. (1) to get an equation involving $f_L, \theta_L, f_R, \theta_R$

Solution 1.1.

1.2. Multiply your equation by $e^{-i\theta_L}$ and take the real and imaginary parts

Solution 1.2.

1.3. Identifying f_L^2 with the number of cooper pairs on the left side of the junction, deduce the "current" across the junction.

Solution 1.3.

1.4. By symmetry one can write down the polar versions of the equations from Eq. (2) by just swapping the labels R and L. Write these equations

Solution 1.4.

1.5. Typically γ is large compared to J, and in the equation for $\partial_t \theta$ one usually neglects the term proportional to J. Ignoring these terms, your equations of motion should look like a pendulum. Linearize these pendulum equations about their equilibrium, and find the Josephson oscillation frequency.

Solution 1.5.