PHYS 3317: Variational Method

Hand in at beginning of next lecture

For these problems you should use the following facts

$$\int_0^\infty dx\, e^{-\alpha x} = 1/\alpha \tag{1}$$

$$\int_0^\infty dx\, x e^{-\alpha x} = 1/\alpha^2 \tag{2}$$

$$\int_0^\infty dx\, x^2 e^{-\alpha x} = 2/\alpha^3. \tag{3}$$

We are going to use the variational method to estimate the zero point energy of the harmonic oscillator. We will minimize

$$E = \int dx \left[-\frac{\hbar^2}{2m}\psi^*(x)\psi''(x) + \frac{1}{2}m\omega^2 x^2 \psi^*(x)\psi(x) \right]. \tag{4}$$

Life is a bit easier if we integrate by parts and write

$$E = \int dx \left[\frac{\hbar^2}{2m}\psi'^*(x)\psi'(x) + \frac{1}{2}m\omega^2 x^2 \psi^*(x)\psi(x) \right]. \tag{5}$$

We will use the ansatz

$$\psi(x) = \frac{e^{-|x|/(2d)}}{\sqrt{2d}}. \tag{6}$$

Problem 1. Calculate $\int |\psi(x)|^2\, dx$.

Solution 1.1.

Problem 2. Calculate $\int \psi'^{*}(x)\psi'(x)\,dx$.

Solution 2.1.

Problem 3. Calculate $\int x^2|\psi(x)|^2\,dx$.

Solution 3.1.

Problem 4. Put these results together to calculate E.

Solution 4.1.

Problem 5. Minimize E. (It might help to first plot it)

Solution 5.1.