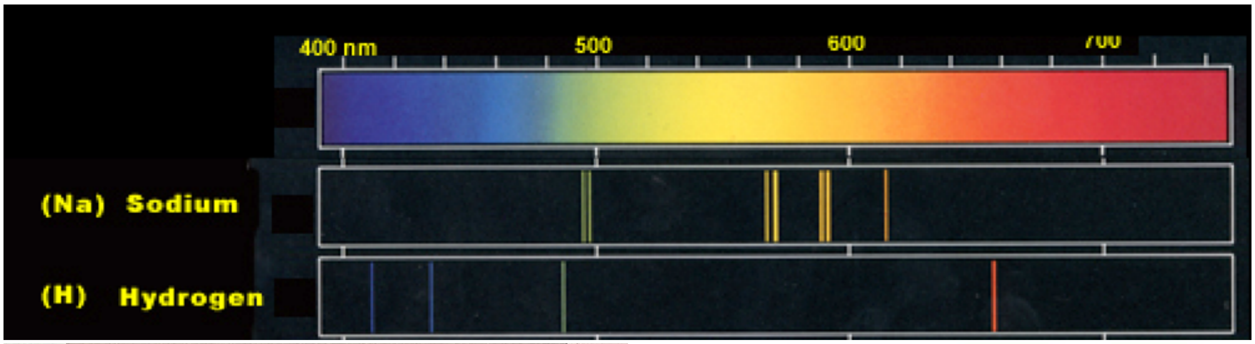
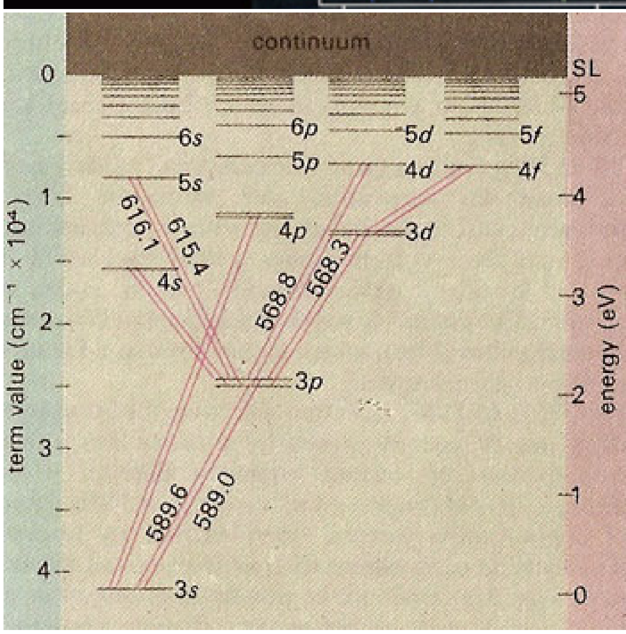
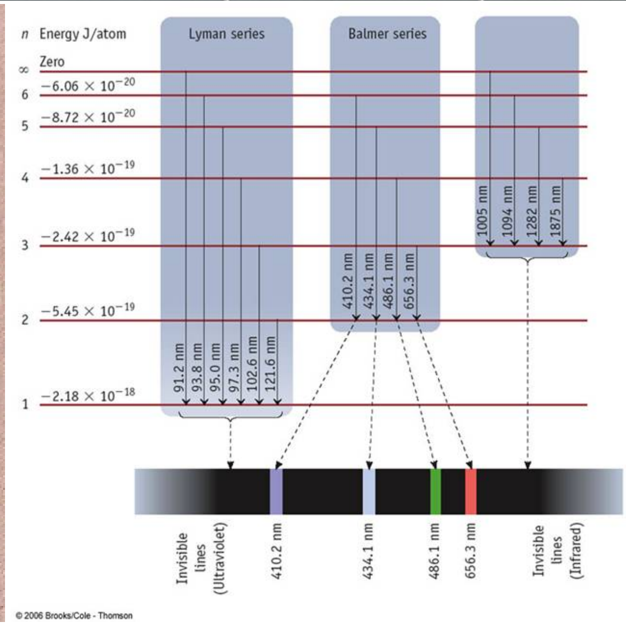
PHYS 3317: Spectra and Symmetries

Hand in at beginning of next lecture







Problem 1. What are the differences between the spectrum of Hydrogen, and that of Helium?

Solution 1.1.

Problem 2. Consider the Hamiltonian for a one-dimensional Schrodinger equation

$$\hat{H} = -\frac{\hbar^2}{2m}\partial_x^2 + V(x). \tag{1}$$

The momentum operator $\hat{p}$ is defined by

$$\hat{p} = -i\hbar\partial_x. \tag{2}$$

As in previous exercises, we make operator identities by acting on a wavefunction. For example, if we denote $\hat{V}$ as the operator which multiplies a wavefunction by $V(x)$,

$$\begin{aligned} \hat{p}\hat{V}\psi(x) &= -i\hbar\partial_x \left[V(x)\psi(x)\right] && (3) \\ &= -i\hbar \left[V'(x) + \partial_x\right]\psi(x). && (4) \end{aligned}$$

This result could be written

$$\hat{p}\hat{V} = -i\hbar\hat{V}' + \hat{V}\hat{p}, \tag{5}$$

where $\hat{V}'$ is the operator that multiplies a wavefunction by $V'(x)$.

2.1. Using this result, what is the commutator $[\hat{p}, \hat{V}]$?

Solution 2.1.

2.2. For what $V(x)$ does $[\hat{p}, \hat{V}]$=0?

Solution 2.2.

2.3. Under what circumstances will $[\hat{p}, \hat{H}] = 0$?

Solution 2.3.

2.4. How does the symmetry of translational invariance relate to $[\hat{p}, \hat{H}] = 0$?

Solution 2.4.

Problem 3. Consider the Hamiltonian for a three-dimensional Schrodinger equation, written in spherical coordinates

$$\hat{H}\psi(x) = -\frac{\hbar^2}{2m}\left[\frac{1}{r^2}\partial_r\left(r^2\partial_r\psi\right) + \frac{1}{r^2\sin\theta}\partial_\theta\left((\sin\theta)\partial_\theta\psi\right) + \frac{1}{r^2\sin^2\theta}\partial_\phi^2\psi\right] + V(r,\theta,\phi) \tag{6}$$

The z-component of the angular-momentum operator $\hat{L}z$ is defined by

$$\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x = \frac{\hbar}{i}\partial_\phi. \tag{7}$$

3.1. By analogy with your one-dimensional calculation, what are the conditions on $V(r,\theta,\phi)$ such that $[\hat{L}_z, \hat{H}] = 0$?

Solution 3.1.

3.2. What symmetry does this condition correspond to?

Solution 3.2.