# PHYS 3317: Occupation numbers

**Tuwsday October 23, 2018**

**Hand in at beginning of next lecture**

**Problem 1. Occupation of an impurity state in silicon** Suppose we have a piece of silicon with an impurity in it. This might be a chip inside your cell phone. Suppose the impurity has a potential which can trap an electron. We want to know how likely it is to have an electron bound there.

**1.1.** The electrons in the chip provide a bath with some chemical potential $\mu$. The impurity state has some energy $\epsilon$. There are two states of our impurity, occupied, and unoccupied. The occupied state has energy $\epsilon$ and number of particles 1. In terms of these quantities, and the partition function $\mathcal{Z}$, what is the probability of having a single electron on the impurity?

**Solution 1.1.**

**1.2.** The unoccupied state has zero energy and zero particles. What is the probability of having a no electrons on the impurity?

**Solution 1.2.**

**1.3.** The partition function is defined so that the total probability is 1. What is $\mathcal{Z}$?

**Solution 1.3.**

**1.4.** What is the average number of particles on the impurity?

**Solution 1.4.**

**Problem 2. Occupation of an impurity state in liquid Helium**

Suppose we instead had an attractive potential on the wall of a container of liquid helium. Imagine the impurity state again has some energy $\epsilon$. Since these are bosons you could put more than one of them on the site. If we neglect interactions, the energy of having $n$ atoms in the state is $n\epsilon$.

**2.1.** In terms of $\epsilon$, $\mu$, $T$, and $\mathcal{Z}$, what is the probability of having $n$ atoms occupying the state?

**Solution 2.1.**

**2.2.** What is $\mathcal{Z}$?

**Solution 2.2.**

**2.3.** What is the average number of particles on the impurity?

**Solution 2.3.**

These two results [2.3 and 1.4] are **really** important, as the results hold for *any* state – not just an impurity. Of course, it neglects interaction – but this is *the* way that one typically starts modeling a system of many particles. For example, in a gas of Rubidium atoms in an atomic trap, one expects that the number of particles with momentum $k$ is

$$n_k = \frac{1}{e^{-\beta(k^2/2m-\mu)} - 1}. \tag{1}$$

You can use this to calculate all of the thermodynamic quantities (pressure, density, energy density...)