PHYS 3317: Phonons C

Thursday Nov 30, 2014

Hand in at beginning of next lecture (only do handout A, B, or C, not all three)

Problem 1. γ-potential Energy

1.1. We defined creation and annihilation operators via

$$x_j = \frac{d}{\sqrt{2}}\left([(a_j + a_j^\dagger) + \alpha(a_{j+1} + a_{j+1}^\dagger + a_{j-1} + a_{j-1}^\dagger)\right] \quad (1)$$

$$p_j = \frac{\hbar}{\sqrt{2}di}\left([(a_j - a_j^\dagger) - \alpha(a_{j+1} - a_{j+1}^\dagger + a_{j-1} - a_{j-1}^\dagger)\right]. \quad (2)$$

where α is small. To leading order in α, these obey

$$[a_i, a_j] = 0 \quad (3)$$

$$[a_i, a_j^\dagger] = \delta_{ij} \quad (4)$$

where δ_{ij} is the Kronecker delta, equal to zero if $i \neq j$ and 1 if $i = j$.

Write the potential energy from the γ oscillators in terms of these creation and annihilation operators. Neglect terms which are of linear or higher order in α.

$$V_\gamma = \sum_j \frac{\gamma}{2}(x_j - x_{j-1})^2 \quad (5)$$

Rewrite your expression in the form:

$$V_\gamma = \sum_j A_\gamma(a_j^\dagger a_j + a_j a_j^\dagger) + B_\gamma(a_{j+1}^\dagger a_j + a_j^\dagger a_{j+1}) + C_\gamma(a_j a_j + a_j^\dagger a_j^\dagger) + D_\gamma(a_{j+1} a_j + a_j^\dagger a_{j+1}^\dagger) \quad (6)$$

Note, to put it in this form, you will need to use the identity:

$$\sum_j c_{j-1} d_j = \sum_j c_j d_{j+1}. \quad (7)$$