PHYS 3317: linear Operators

Thursday Aug 23, 2018

Hand in at beginning of next lecture

Problem 1. Quantum Operators

In Quantum Mechanics, physical observables are associated by operators $\hat{O}$ which act on the wavefunctions $\psi(x)$. For example, the operator $\hat{x}$ is defined by

$$(\hat{x}\psi)(x) = x\psi(x).$$

Note, we use the same symbol here for the operator and the argument of the wavefunction. This is confusing. These operators are *linear*, meaning that for any two wavefunctions $\psi(x)$ and $\phi(x)$, one must have $\hat{O}(a\psi + b\phi) = a\hat{O}\psi + b\hat{O}\phi$. Prove that $\hat{x}$ is linear.

Solution 1.1.

Problem 2. What (real space) wavefunction is an eigenvector of $\hat{x}$ with eigenvalue x_0?

Solution 2.1.

Problem 3. Momentum Operators

The momentum operator is defined by

$$(\hat{p}\psi)(x) = -i\hbar\psi'(x).$$

Prove that the momentum operator is linear.

Solution 3.1.

Problem 4. What (real space) wavefunction is an eigenvector of $\hat{p}$ with eigenvalue p_0?

Solution 4.1.