PHYS 3317: Maxwell's Equations in 2+1D

Hand in at beginning of next lecture

Problem 1. Constituent Equation

In 2+1D electromagnetism there are three physical fields

$$S_t = F_{xy} = \partial_x A_y - \partial_y A_x \tag{1}$$

$$S_y = F_{tx} = \partial_t A_x - \partial_x A_t \tag{2}$$

$$S_x = F_{yt} = \partial_y A_t - \partial_t A_y. \tag{3}$$

Show that

$$\partial_t S_t + \partial_x S_x + \partial_y S_y = 0, \tag{4}$$

Solution 1.1.

Problem 2. Light in 2D

2.1. In the absence of matter, the 2D Maxwell equations read

$$\partial_x S_y - \partial_y S_x = 0 \tag{5}$$

$$-\partial_y S_t - \partial_t S_y = 0 \tag{6}$$

$$\partial_t S_x + \partial_x S_t = 0 \tag{7}$$

$$\partial_t S_t + \partial_x S_x + \partial_y S_y = 0, \tag{8}$$

Make the ansatz

$$S_\nu(x,y,t) = S_\nu e^{-i\omega t + ik_x x + ik_y y} \tag{9}$$

to find the dispersion relationship.

Solution 2.1.

2.2. For a given $\vec{\mathbf{k}}$, how many modes are there? What is the physical interpretation?

Solution 2.2.

2.3. A somewhat more complicated way to do the same calculation is to work with the potentials A_x, A_y, A_t. This more complicated approach will come in handy on Thursday. Write Eq. (5) in terms of A.

Solution 2.3.

2.4. Make the ansatz

$$A_\nu(x, y, t) = A_\nu e^{-i\omega t + ik_x x + Ik_y y}, \tag{10}$$

and fill in the boxes below.

Solution 2.4. .

$$\begin{pmatrix} \omega^2 - k_y^2 & \square & \square \\ \square & \omega^2 - k_x^2 & \square \\ \square & \square & k_x^2 + k_y^2 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_t \end{pmatrix} = 0$$

2.5. Show that

$$\begin{pmatrix} -k_y \\ k_x \\ 0 \end{pmatrix} \tag{11}$$

is an eigenvector of this matrix. What is its eigenvalue?

Solution 2.5.

2.6. Can this eigenvalue ever be zero? Under what circumstances? Give a physical interpretation.

Solution 2.6.

2.7. Show that

$$\begin{pmatrix} k_x \\ k_y \\ -\omega \end{pmatrix} \tag{12}$$

is an eigenvector of this matrix. What is its eigenvalue?

Solution 2.7.

2.8. Can this eigenvalue ever be zero? Under what circumstances? Give a physical interpretation.

Solution 2.8.

2.9. Show that

$$\begin{pmatrix} k_x\omega \\ k_y\omega \\ k^2 \end{pmatrix} \tag{13}$$

is an eigenvector of this matrix. What is its eigenvalue?

Solution 2.9.

2.10. Can this eigenvalue ever be zero (for real ω)?

Solution 2.10.