PHYS 3317: Higgs Mechanism

Thursday November 13, 2014

Hand in at beginning of next lecture

The Higgs mechanism occurs when we flip the sign of the mass term in the Klein-Gordon equation: taking

$$(\partial_t - iA_t)^2\phi - (\partial_x - iA_x)^2\phi - (\partial_y - iA_y)^2\phi - m^2\phi + \lambda^2|\phi|^2\phi = 0. \tag{1}$$

This is coupled to Maxwell's equations:

$$\partial_t[\partial_x A_x + \partial_y A_y] - (\partial_x^2 + \partial_y^2)A_t = \frac{\phi^*\partial_t\phi - \phi\partial_t\phi^*}{2i} - |\phi|^2 A_t \tag{2}$$

$$\partial_x[-\partial_y A_y + \partial_t A_t] - (-\partial_y^2 + \partial_t^2)A_x = -\frac{\phi^*\partial_x\phi - \phi\partial_x\phi^*}{2i} + |\phi|^2 A_x \tag{3}$$

$$\partial_y[\partial_t A_t - \partial_x A_x] - (\partial_t^2 - \partial_x^2)A_y = -\frac{\phi^*\partial_y\phi - \phi\partial_y\phi^*}{2i} + |\phi|^2 A_y \tag{4}$$

Problem 1. Linearizing the Matter equations

1.1. Find a stationary solution to Eq. (1) of the form $\phi(x,y,t) = \phi_0$ and $A_x = A_y = A_t = 0$, where ϕ_0 is uniform in space and time. Relate ϕ_0 to the parameters in these equations.

Solution 1.1.

1.2. Linearize Eq. (1), writing $\phi(x, y, t) = \phi_0 + a(x, y, t) + ib(x, y, t)$. Treat a, b, A_x, A_y, A_t as small – discarding all terms which are quadratic in them.

Solution 1.2.

1.3. Take the real and imaginary parts of your expression, finding to separate real differential equation.

Solution 1.3.

1.4. The equation for a should separate. Find the dispersion of this mode by making the ansatz $a(x, y, t) = ae{-}i\omega t + ik_x x + ik_y y$, and solve for ω as a function of k. This is the "Higgs mode." It corresponds to the massive particle (the Higgs boson) which was found at Cern.

Solution 1.4.

1.5. What is the mass of the Higgs boson in terms of the parameters of the theory.

Solution 1.5.

Problem 2. Light coupled to a condensate

2.1. Linearize Maxwell's equations, Eq. (2).

Solution 2.1.

2.2. Make the Ansatz $b(x,y,t) = be^{-i\omega t+ik_x+ik_y}$ and $A_\nu(x,y,t) = A_\nu e^{-i\omega t+ik_x x+Ik_y y}$. Fill in the boxes in the matrix

Solution 2.2. .

$$\begin{pmatrix} \omega^2 - k_y^2 & \Box & \Box & ik_x\phi_0 \\ \Box & \omega^2 - k_x^2 & \Box & ik_y\phi_0 \\ \Box & \Box & k_x^2 + k_y^2 & i\omega\phi_0 \\ -ik_x\phi_0 & -ik_y\phi_0 & -i\omega\phi_0 & \Box \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_t \\ b \end{pmatrix} = 0$$

2.3. Show that

$$\begin{pmatrix} -k_y \\ k_x \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

is an eigenvector of this matrix. What is its eigenvalue?

Solution 2.3.

2.4. Can this eigenvalue ever be zero? Under what circumstances? Give a physical interpretation. The photon (W boson) now has mass.

Solution 2.4.

2.5. In terms of parameters of the model, what is the mass of the W?

Solution 2.5.

2.6. Is there a relationship between the mass of the W and the mass of the Higgs?

Solution 2.6.

2.7. Show that

$$\begin{pmatrix} \phi_0 k_y \\ \phi_0 k_x \\ 0 \\ 2ik_x k_y \end{pmatrix} \tag{6}$$

is an eigenvector of this matrix. What is its eigenvalue?

Solution 2.7.

2.8. Can this eigenvalue ever be zero? Under what circumstances? This can be thought of as a second massive photon – but this one is a longitudinal mode, rather than a transverse mode. Maxwell's equations in free space do not support such a mode, but coupling to the Higgs field introduced its possibility. Interestingly, it has the same dispersion as the regular transverse photon mode.

Solution 2.8.

2.9. Show that

$$\begin{pmatrix} k_x\omega \\ k_y\omega \\ k^2+\phi_0^2 \\ -i\phi_0\omega \end{pmatrix} \tag{7}$$

is an eigenvector of this matrix. What is its eigenvalue?

Solution 2.9.

2.10. Can this eigenvalue ever be zero (for real ω)?

Solution 2.10.

2.11. Show that

$$\begin{pmatrix} k_x \\ k_y \\ -\omega \\ -i\phi_0 \end{pmatrix} \tag{8}$$

is an eigenvector of this matrix. What is its eigenvalue?

Solution 2.11.

2.12. Can this eigenvalue ever be zero? Under what circumstances? Give a physical interpretation.

Solution 2.12.