



PHYS 3317: Commutation Relationships

Thursday Aug 30, 2018

Hand in at beginning of next lecture

Problem 1. Operator identities.

We say that two operators A and B are the same if for every function $\psi(x)$

$$(A\psi)(x) = (B\psi)(x). \quad (1)$$

1.1. Prove that $[x, p] = xp - px = i\hbar$, where $p = (\hbar/i)\partial_x$ is the momentum operator.

Solution 1.1.

Operator identities like this simplify calculations.

1.2. Calculate $[x, p^2]$.

Solution 1.2.



Problem 2. Commutator identities.

2.1. Prove that $[A, BC] = B[A, C] + [A, B]C$

Solution 2.1.

2.2. Prove that $[AB, C] = A[B, C] + [A, C]B$

Solution 2.2.

Problem 3. Bonus Problem. (Do on separate page – just for fun, no credit)

3.1. Use Taylor's theorem to prove that for any constant a ,

$$e^{a\partial_x} f(x) = f(x + a), \quad (2)$$

or equivalently

$$e^{iap} f(x) = f(x + a). \quad (3)$$

3.2. Use this result to show that

$$[e^{iap}, x] = ae^{iap} \quad (4)$$