PHYS 3317: Amplitude and Phase

Thursday Aug. 30, 2018

Hand in at beginning of next lecture

In this handout you will explore a *fluid mechanics* analogy to the time dependent Schrodinger equation

$$i\hbar\partial_t\psi(x,t) = -\frac{\hbar^2}{2m}\partial_x^2\psi(x,t). \tag{1}$$

We will write the complex wavefunction as an amplitude and a phase

$$\psi(x,t) = f(x,t)e^{i\phi(x,t)}, \tag{2}$$

where both f and ϕ are real. Physically $f^2(r,t)$ is the probability density for finding the particle at position x at time t. As you will see, $(\hbar/m)\partial_x\phi$ can be interpreted as the average velocity of the particle if it is near position x at time t.

Problem 1. What are the dimensions of f^2?

Solution 1.1.

Problem 2. What are the dimensions of ϕ?

Solution 2.1.

Problem 3. In terms of $\partial_t f$ and $\partial_t \phi$, what is $\partial_t \psi$?

Solution 3.1.

Problem 4. In terms of $\partial_x f$ and $\partial_x \phi$, what is $\partial_x \psi$?

Reuse your previous result. [Don't make work for yourself by repeating something you have already done.]

Solution 4.1.

Problem 5. In terms of $\partial_x f$ and $\partial_x \phi$, what is $\partial_x^2 \psi$?

Solution 5.1.

Problem 6. Use these results to write the time dependent Schrodinger equation in terms of f and ϕ.

Solution 6.1.

Problem 7. Multiply this equation by $e^{-i\phi}$, and take the imaginary part to find an equation of the form

$$\partial_t f = I(f, \partial_x f, \partial_x \phi, \partial_x^2 \phi). \tag{3}$$

Find the function I.

Solution 7.1.

Problem 8. Multiply this equation by $2f$, and use the identities

$$2f\partial_t f = \partial_t f^2 \quad (4)$$
$$2f\partial_x f = \partial_x f^2 \quad (5)$$

to write an equation of the form

$$\partial_t f^2 = \tilde{I}(f^2, \partial_x f^2, \partial_x \phi, \partial_x^2 \phi). \quad (6)$$

Find $\tilde{I}$.

Solution 8.1.

Problem 9. Show that $\tilde{I}$ is a total derivative

$$\partial_t f^2 = \tilde{I} = -\partial_x j. \quad (7)$$

This is known as the continuity equation. Find j. Physically this means that the rate of change of the probability density is given by the difference between the flow in and the flow out.

Solution 9.1.

Problem 10. What are the dimensions of j?
From the continuity equation, j corresponds to the amount of probability density per unit time that flows through some surface placed at x.

Solution 10.1.

Problem 11. What is the velocity of the flow of probability?
Given that you know the probability flux, and the probability density, you should be able to argue what the velocity is.

Solution 11.1.