PHYS 3317: Dynamics

Tuesday Sept 4, 2018

Hand in at beginning of next lecture

In a simplified model of ammonia, the nitrogen atom can be in one of two places. The amplitude that it is on the left is ψ_L. The amplitude that it is on the right is ψ_R. We argued by symmetry that the equations of motion had to be of the form

$$i\partial_t \begin{pmatrix} \psi_L(t) \\ \psi_R(t) \end{pmatrix} = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \begin{pmatrix} \psi_L(t) \\ \psi_R(t) \end{pmatrix}. \tag{1}$$

We want to solve this differential equation with the initial conditions $\psi_L(0) = 1$ and $\psi_R(0) = 0$.

Problem 1.

1.1. The eigenvectors of the matrix

$$H = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \tag{2}$$

are

$$|+\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}, \qquad |-\rangle = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}. \tag{3}$$

What are the eigenvalues E_+ and E_-? [Note, the notation $|+\rangle$ is a common way to denote a column vector in an abstract state space. Physicist like to reserve the notations $\vec{v}$ or $\mathbf{v}$ for three dimensional vectors.]

Solution 1.1.

Problem 2. We can write

$$|\psi(t)\rangle = \begin{pmatrix} \psi_L(t) \\ \psi_R(t) \end{pmatrix} = \alpha(t)|+\rangle + \beta(t)|-\rangle. \tag{4}$$

2.1. Prove that if $\alpha|+\rangle + \beta|-\rangle = \gamma|+\rangle + \delta|-\rangle$, then $\alpha = \gamma$ and $\beta = \delta$. This is known as the uniqueness of the eigenstate expansion.

Solution 2.1.

2.2. At time $t = 0$, the nitrogen atom is on the left: $\psi_L(0) = 1$ and $\psi_R(0) = 0$. Find $a(0)$ and $b(0)$.

Solution 2.2.

Problem 3.

3.1. Substitute Eq. (4) this into Eq. 1, and use the uniqueness of the eigenstate expansion, to find differential equations for $a(t)$ and $b(t)$.

Solution 3.1.

3.2. Solve these differential equations, and use the initial conditions to find $a(t)$ and $b(t)$.

Solution 3.2.

Problem 4.

4.1. What is the probability that at time t the Nitrogen atom is on the right?

Solution 4.1.