PHYS 3317: Stimulated Emission

Hand in at beginning of next lecture

Problem 1. Integrating the Linearized Equations
Consider an ammonia atom. The amplitude of being in the ground state with energy $E_s = -\Delta$ is $a(t)$, while the amplitude of being in the excited state with energy $E_a = \Delta$ is $b(t)$. At time $t = 0$, $a = 0$ and $b = 1$. We argued that in the presence of an oscillatory electric field, the equations of motion for these coefficients are

$$i\hbar\partial_t \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} E_s & \epsilon\cos(\nu t) \\ \epsilon\cos(\nu t) & E_a \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}, \tag{1}$$

where $2\epsilon\cos(\nu t)$ is the energy difference between the left and right states induced by the oscillating electromagnetic field. We will solve this assuming ϵ is small. We will assume that the solutions to this equation can be expressed as a Taylor series in ϵ,

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a^{(0)} \\ b^{(0)} \end{pmatrix} + \epsilon \begin{pmatrix} \delta a \\ \delta b \end{pmatrix} + \cdots \tag{2}$$

where the neglected terms are higher order in ϵ. Our initial conditions read $a^{(0)}(t = 0) = 0$, $\delta a(t = 0) = 0$, $b^{(0)}(t = 0) = 1$, $\delta b(t = 0) = 0$.

1.1. Substitute Eq. (2) into Eq. (1), and collect the terms which are independent of ϵ, and the terms which are linear in ϵ.

Solution 1.1.

1.2. Solve the ϵ independent equations to find $a^{(0)}(t)$ and $b^{(0)}(t)$

Solution 1.2.

1.3. Substitute these "zero-th order" solutions into the the first order equations to show that

$$i\hbar\partial_t \delta a(t) = E_s \delta a(t) + \epsilon \cos(\nu t) e^{-iE_a t/\hbar}. \tag{3}$$

Solution 1.3.

1.4. Show that the solution to Eq. (3) is

$$a(t) = A[e^{i(\hbar\nu - E_a)t/\hbar} - e^{-iE_s t/\hbar}] + B[e^{-i(\hbar\nu + E_a)t/\hbar} - e^{-iE_s t/\hbar}], \tag{4}$$

and find A and B.

Solution 1.4.

1.5. We define the detuning $\delta = E_a - E_s - \hbar\nu$. If δ is small compared to $E_a - E_s$, show that one then has $B \gg A$.

Solution 1.5.

1.6. Neglecting the B term, what is the probability of being in the symmetric state as a function of time?

Solution 1.6.