



## PHYS 3317: Two-level systems

Hand in at beginning of next lecture

Consider three Hermitian operators corresponding to possible measurements

$$S_z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (1)$$

$$S_{120} = \frac{1}{4} \begin{pmatrix} -1 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}. \quad (2)$$

$$S_{-120} = \frac{1}{4} \begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}. \quad (3)$$

Their eigenvalues are  $\pm 1/2$ , with eigenvectors

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (4)$$

$$|\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (5)$$

$$|\uparrow 120^\circ\rangle = \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \end{pmatrix} \quad (6)$$

$$|\downarrow 120^\circ\rangle = \begin{pmatrix} \sqrt{3}/2 \\ -1/2 \end{pmatrix} \quad (7)$$

$$|\uparrow -120^\circ\rangle = \begin{pmatrix} 1/2 \\ -\sqrt{3}/2 \end{pmatrix} \quad (8)$$

$$|\downarrow -120^\circ\rangle = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}. \quad (9)$$



**Problem 1.**

**1.1.** Suppose I prepare a two-level system in state  $|\uparrow\rangle$  I then measure it at  $120^\circ$ . What possible results can I find, and with what probabilities?

**Solution 1.1.**

**1.2.** Suppose I prepare a two-level system in state  $|\uparrow 120^\circ\rangle$  I then measure it at  $-120^\circ$ . What possible results can I find, and with what probabilities?

**Solution 1.2.**